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## M DWARFS: THEORETICAL WORK

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## MODEL PHOTOSPHERES

Theoretical work on the atmospheres of M dwarfs has progressed along lines parallel to those followed in the study of other classes of stars. The earliest work was aimed solely at constructing models of the photosphere, in which the only relevant energy fluxes were considered to be thermal (radiative plus convective). Such models have become increasingly sophisticated as improvements in opacities, in the equation of state, and in the treatment of convection have been incorporated during the last 15 to 20 years. As a result, spectrophotometric data on M dwarfs can now be fitted rather well by current models. The section *Thermal Model Atmospheres of Normal M Dwarfs* summarizes the various attempts at modeling M dwarf photospheres in purely thermal terms. Some extensions of these models to include (in an ad hoc manner) the effects of microturbulence and magnetic inhomogeneities are summarized in the sections *Quasi-Thermal Model Photospheres of dM Stars* and *Quasi-Thermal Model Photospheres of dMe Stars*. The thermal models can be constrained not only by predictions of *emergent* radiation, but also by carrying the integrations *inward* and constructing credible models of the entire star. In this regard, the M dwarfs are in a somewhat unique position because they bracket the mass range in which main-sequence stars are believed to be-

come completely convective. Slight errors in surface parameters and in modeling techniques can lead to large uncertainties in the interior structure (because one picks out the wrong adiabat near the surface). Thus, M dwarfs may allow one to check the constitutive thermal physics which enter into the study of all stars. These points are summarized in the section *Thermal Models of Internal Structure*, which is aimed at placing atmospheric studies in perspective. The chromospheric and coronal phenomena have dominated the literature on M dwarfs so much in recent years that one is tempted to lose sight of the ultimate origin of the energy which makes these phenomena possible: nuclear processes in the deep interior. Although the interior processes are believed to be entirely thermal in nature and therefore supposedly readily amenable to modeling, it appears that significant uncertainties still persist in our knowledge of the internal structure of M dwarfs. Thus, even the thermal models of M dwarfs require further study.

Despite the successes which the thermal models have had, nonthermal processes also make a contribution to the physics of M dwarf atmospheres. The first flare (in continuum light) in an M dwarf was recognized some 60 years ago, and as data were accumulated on such events, an increasing amount of attention was paid to the similarities between these events and transient brightenings (in chromospheric

lines) in the Sun. The latter represent the most spectacular form of nonthermal energy release in the Sun. Of course, other evidence for the presence of nonthermal energy in the solar atmosphere is provided by the presence of a corona and chromosphere even outside flares. In fact, the question of where one draws a dividing line between flares and coronal heating in the Sun has received much attention recently (Lin et al., 1984; Porter et al., 1984), and no definitive answer is presently available. Precisely the same question arises in the M dwarfs (Doyle and Butler, 1985; Butler and Rodonò, 1985): their prominent flare activity is not the only evidence of nonthermal processes in the atmosphere. During even the earliest years of stellar spectroscopic classification, certain M dwarfs distinguished themselves from most other cool stars by the presence of prominent emission lines of hydrogen. (Emission in the cores of the calcium lines was also known, but this is an almost ubiquitous feature of cool stars.) The Balmer emission lines provided the first clear sign that modeling the atmospheres of all M dwarfs in terms of purely thermal energy fluxes would be inadequate; at least a chromosphere would have to be provided to account for the hydrogen emission. In order to model the nonthermal processes, the first step is to determine the requirements of nonthermal energy fluxes in the chromosphere; this process has gotten under way in the last few years. (See the section *Semiempirical Chromospheric Models*.) The second step is harder: to identify the source of the nonthermal energy and model it in physically realistic ways. This step has not yet been solved, although a surprising recent development is of major interest in this regard. X-ray data from the Einstein satellite have indicated that many M dwarfs emit large fluxes of X rays; in fact, when expressed as fractions of the bolometric flux, the M dwarfs emit more X rays than any other cool stars. (See the section *Semiempirical Transition Region and Coronal Models*.) In fact, the *corona* in an M dwarf may be supplied with even more nonthermal energy than the chromosphere in that star. Astronomers who are interested in bolometric

magnitudes with an accuracy of 0.01 magnitude must now include the X-ray emission from the corona if they are to achieve such precision in the case of many M dwarfs. (See the section *Nonthermal Models of M Dwarf Atmospheres*.) The strong coronae in M dwarfs, in which flaring activity is also pronounced, have once again raised the question of whether or not a dividing line exists between coronal heating and flaring (Mullan, 1979; Doyle and Butler, 1985; Butler and Rodonò, 1985). Models for flares in M dwarfs are presently available only in outline form. (See the section *Semiempirical Flare Models*.)

### Thermal Model Atmospheres of Normal M Dwarfs

The combination of high gravity and low effective temperatures in red dwarfs ensures copious molecular formation. By "copious," we mean that the number density of molecules (eg.,  $H_2$ ) becomes a significant fraction of the atomic number density (e.g.,  $H$ ), or even exceeds it at certain levels in the atmosphere (see Figure 10-1). With the new degree of freedom associated with the molecules, the equation of state is altered. Thus, as well as the well-known convective instability associated with the ionization of abundant *atomic* species deep in the photosphere ( $\tau_{5000} \gtrsim 1$ ), dissociation of abundant *molecules* (especially  $H_2$ ) creates a new convective instability at rather high levels of the photosphere ( $\tau_{5000} < 0.1$ ). Historically, model atmospheres of M dwarfs have evolved in the direction of attempting to incorporate the double complexities of molecule formation and optically thin convection with increasing realism. Dust may also form in the coolest stars if no chromosphere is present. Molecules and dust have an especially serious effect in the opacities, increasing them by up to  $10^5$  relative to the purely atomic values (Bohn, 1981).

The earliest thermal model atmosphere of an M dwarf (Tsuji, 1966) ignored convection altogether, but included opacities due to  $H_2O$ ,  $H_2$ , and CO. Vardya (1966) treated convection, using the mixing-length formalism of

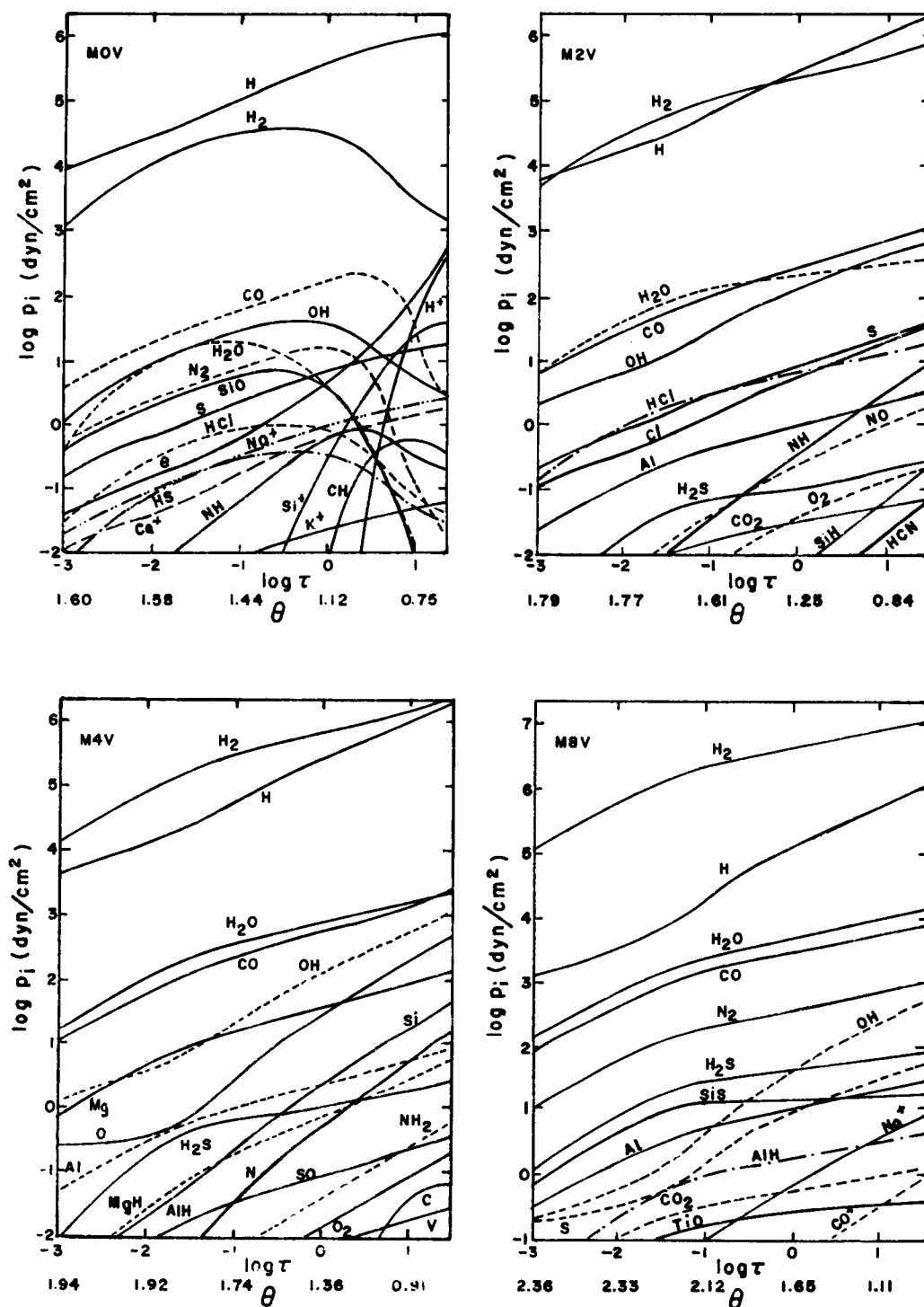


Figure 10-1. Variation of relative abundances of molecules as a function of depth in the model atmospheres of Vardya (1966). Abscissa scales are  $\log$  (continuum optical depth) and  $\theta = 5040/T$ . Ordinates are partial pressures of various species. Results for M0, M2, M4, and M8 dwarfs are shown for selected molecular species.

Heney et al. (1965), and derived thermal models in which the molecular equilibrium of 160 species (including TiO, CaH, MgH) were subsequently evaluated (see Figure 10-1). Vardya felt that it would be premature to attempt to iterate the convective/molecular equations (including molecular opacities) because of deficiencies in, for example, the convection treatment and in "missing opacity" (Vardya and Böhm, 1965). Heavy line blanketing by overlapping lines is difficult to incorporate when so many different molecular species are present in the atmosphere. Thus, it is almost inevitable that one misses some important sources of opacity. Nevertheless, Vardya's results are important in that they suggest that molecular hydrogen is the dominant species throughout the atmosphere (rather than atomic hydrogen) at spectral types M4 and later ( $T_{\text{eff}} < 3230$  K).

Kandel (1967a, 1967b) included even more molecular species than Vardya (1966). Kandel was concerned about the presence of a density inversion in some earlier model photospheres: he thought this might be attributed to deficiencies in the standard mixing-length formalism of convection (in which the mixing-length parameter  $\alpha = L/H_p$  is kept constant at all depths). He therefore devised a modified mixing-length formalism in which the parameter  $\alpha$  is allowed to vary with depth in such a way that the density inversion is just eliminated. (This is Kandel's "minimum convection condition.") He found that  $\alpha$  must vary greatly with depth (by factors of  $10^4$ ). Radiative fluxes from the models were found to fit R-I colors well, but predicted colors at shorter and longer wavelengths became increasingly poor, presumably due to lack of major sources of line opacity. In recent years, it has become apparent that the presence of a density inversion in a model may have little or nothing to do with errors in the mixing-length theory: a detailed 2-D treatment of the Navier-Stokes equation shows that a density inversion is a natural feature of a convection zone where hydrogen is ionizing, at least in the warmer stars, A5 and F0 (Chan and Sofia, 1984).

A better treatment of H<sub>2</sub>O line opacity (due to Auman) allowed Hershey (1968) to obtain more realistic emergent flux distributions from his models: Hershey's predicted continua show radical departures from Planck behavior around  $\lambda = 1 \mu$ . To obtain flux-constant models, Hershey used the Avrett-Krook temperature correction method, but with the important addition of convection (with constant  $\alpha = 1$ ). Inclusion of convection had the computational effect that convergence to flux constancy was considerably more difficult to achieve than in the purely radiative case. In the coolest models ( $T_{\text{eff}} = 3200$  K), convection was found to carry 10 percent of the flux already at very high levels in the photosphere ( $\tau = 0.05$ ). Hershey concluded that, because of the efficiency of convection even in the high photosphere, it is not permissible to scale the  $T(\tau)$  relation from the Sun to the stars with  $T_{\text{eff}} < 4000$  K. Hershey used his models to compute the wings of strong lines (Ca I 4226, Na D, H $\alpha$ , Mg Ib, Ca H + K) and was successful with H $\alpha$  and Na D. (Notice that only the wings can be computed reliably by these purely photospheric models.) However, he also found (cf. Vardya and Böhm, 1965) that, at  $\lambda 4226 \text{ \AA}$ , there must be "missing opacity" which is several times larger than the known continuous opacity.

Auman (1969) applied his extensive H<sub>2</sub>O opacity calculations to cool stars. He devised a method of replacing a large number of closely spaced lines by a representative mean opacity; this was an important step forward in modeling the thermal photospheres of cool dwarfs. Auman found that, in dwarfs, H<sub>2</sub>O is the dominant source of opacity at  $T_{\text{eff}} < 2520$  K. In the equation of state, Auman calculated molecular equilibria for 58 species (not including TiO, although bands of TiO are prominent in M dwarf spectra (cf. Figure 10-2)). In the presence of H<sub>2</sub>O, the surface temperatures fall below the values which would occur (in a star of given  $T_{\text{eff}}$ ) without H<sub>2</sub>O; this occurs because H<sub>2</sub>O opacity reaches its maximum on the redward side of the Planck peak in these stars. Auman commented on the difficulties of

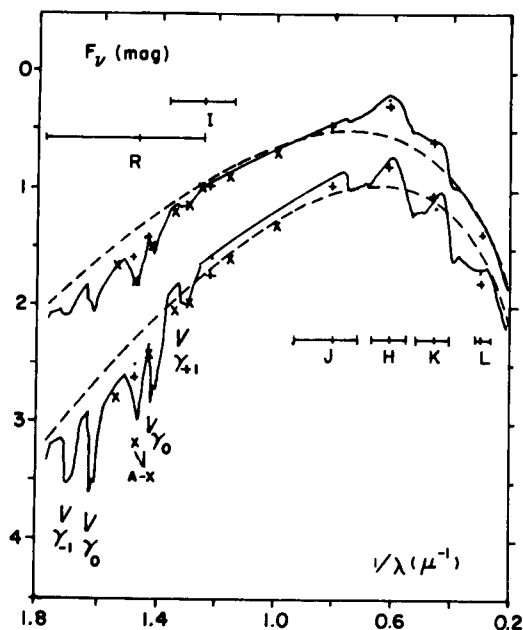


Figure 10-2. Emergent fluxes from models with  $T_{\text{eff}} = 3750$  and  $3250$  K are shown by solid lines (Mould, 1976). Broadband photometry and scans of two stars, Yale 4794 and Yale 3501, are denoted by + and x, Yale 4794 being above. Half-power bandpasses of broadband filters are shown. Also shown are bands of TiO ( $\gamma$  and  $\gamma'$  systems  $\Delta V = 0$  and 1) and CaH. Dashed curves are from blackbody radiators of equal total flux.

applying his model predictions to observed fluxes. The predicted emergent fluxes have been determined to be sensitive to the way in which one derives a mean opacity (i.e., whether one uses a straight mean or a harmonic mean of the  $\sim 10^6$  lines in Auman's case). Moreover, the wavelength dependence of the dominant absorbers ( $\text{H}^-$  and  $\text{H}_2\text{O}$ ) are different, and the predicted spectral shape depends on where the opacities "cross over." In turn, the latter depends on the metal abundances (controlling free electrons) and on the competition of C, Si, and H for the oxygen atoms. Since metal abundances in M dwarfs may differ from solar by 2 to 3 (Mould, 1978), these can affect the atmospheric structure by several hundred degrees (Carbon, 1979). Lack of TiO in these models

makes Auman's colors less accurate in the visible region.

Mould (1976) computed an extensive grid of models combining Tsuji's molecular equilibria (including even more molecular species than Vardya did) with the ATLAS model atmosphere code. By allowing for atomic and molecular line blanketing (which redistributes emergent flux across the spectrum), Mould obtained emergent spectra which could be fitted rather well to RIJHKL filter data for several M dwarfs (see Figure 10-2). In particular, note that the strong TiO bands and  $\text{H}_2\text{O}$  bands appear prominently in the emergent fluxes from Mould's models. By matching many colors of each star to the colors predicted by the models according to certain weighting functions (determined by temperature sensitivity), Mould and Hyland (1976) exploited information contained in many bandpasses, distributed across a broad spectral range, to derive  $T_{\text{eff}}$  values for 20 M dwarfs with an accuracy of  $\pm 100$  K at 4000 K and  $\pm 200$  K at 2850 K. These represent substantial improvements in accuracy compared to deriving  $T_{\text{eff}}$  by fitting a single observed color, say R-I, to the predicted color of a model. A striking colorimetric feature which emerged from Mould's work is shown in Figure 10-3: the J-H color of dwarfs is *not* a monotonic function of the H-K color. Mould showed that this behavior is due to the onset of efficient convection in the upper atmosphere when  $\text{H}_2$  molecules form. (Giants do not show the effect because convection is not efficient enough; radiative leakage between hot and cool streams is more serious in the lower density gas of a giant atmosphere.) The peak J-H color in Figure 10-3 is a measure of metal abundance and gravity. The major source of uncertainty in Mould's models is probably his treatment of water vapor opacities; his values appear to be underestimates of the true opacities (Mould, 1980).

Norlund (1976) applied his two-stream convection model to red dwarfs ( $T_{\text{eff}} = 3750$  and  $4250$  K) and found that the high gas densities result in temperature differences between hot and cold streams which are much less than in

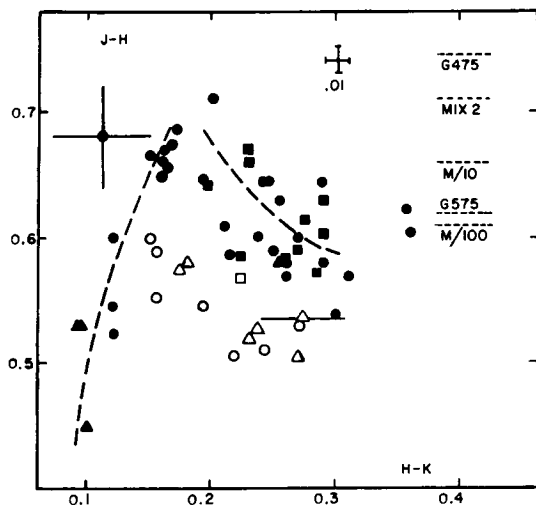


Figure 10-3. Two-color diagram ( $J-H$ ,  $H-K$ ) for red dwarfs in halo and disk populations. Theoretical maximum  $J-H$  values for various models are indicated by horizontal dashed lines at upper right. Models differ in  $\log g$ , mixing length, and metal abundances. Solid circles = old disk stars; solid squares = young disk stars; open symbols = halo stars; solid triangles =  $K$  dwarfs.

the solar case. As a result, predictions of standard mixing-length theories in red dwarfs might be expected to be more reliable than those in the Sun.

#### Quasi-Thermal Model Photospheres of dM Stars

In some of the foregoing models (Auman, Mould), the authors incorporated a nonthermal line broadening (microturbulence) of  $\xi = 2 \text{ km s}^{-1}$ . The reason for this choice is the observed line broadening in red dwarfs ( $0.9$  to  $2.6 \text{ km s}^{-1}$ ; Bonsack and Culver, 1966). Presumably, microturbulence has something to do with convective flows. However, convective velocities,  $v_c$ , in, for example, Auman's models turned out to be quite small ( $\sim 0.3 \text{ km s}^{-1}$ ). The relation between  $v_c$  and  $\xi$  in red dwarfs is therefore obscure. In the Sun, Cloutman (1979) has suggested that the solar granules are not true convective cells, but are bubbles which break away from a density inversion below the

surface. If the same is true in dwarfs (cf. Kandel's (1967a) discussion of density inversions), perhaps this is the source of the discrepancy between  $v_c$  and  $\xi$ . However, it would then become necessary to incorporate turbulent pressure gradients in all hitherto calculated model atmospheres.

A further aspect of the microturbulence is its height dependence: in the Sun,  $\xi = 1$  to  $2 \text{ km s}^{-1}$  in the photosphere, but it increases rapidly upward (reaching  $\sim 10 \text{ km s}^{-1}$  in the chromosphere). On the other hand, in dM and dMe stars, it appears that  $\xi$  remains small ( $< 2 \text{ km s}^{-1}$ ) even in the chromosphere (Giampapa et al., 1982b). This may be due to stronger magnetic fields in the M dwarf chromospheres which restrict the motion of ionized gas.

Uncertainties in understanding the origin of microturbulence have a serious effect when one wishes to predict the acoustic fluxes emitted by a convection zone. (These fluxes may be important in heating chromospheres and/or coronae; see below.) Which velocity parameter,  $v_c$  or  $\xi$ , should be used in the Lighthill-Proudman acoustic formula? Uncertainties of 3 to 10 in the velocity parameter create errors of  $10^4$  to  $10^8$  in the predicted acoustic flux which might heat the chromosphere. Discussions of chromospheric heating along the main sequence have not included these uncertainties (e.g., Bohn, 1981).

#### Quasi-Thermal Model Photospheres of dMe Stars

This section summarizes work which has been done on inhomogeneities and on the modeling of magnetic effects.

Dark spots exist on the surface of certain dMe stars, especially those which rotate faster than a critical velocity ( $\sim 5 \text{ km/s}$ ; Bopp and Espenak, 1977). The spots are cooler than the photosphere by at least several hundred degrees (ensuring quite different molecular equilibria in the spot atmosphere than outside). The spot areas may be  $>10$  percent of the disk area. The problem of energy transport through the star in the presence of such a gross inhomogeneity

is one of great interest (Spruit, 1982): part of the normal energy flux may be trapped beneath the surface, reducing the bolometric luminosity of the star. (This occurs in the solar case; much of the missing flux of large sunspots essentially disappears from the solar "constant." See Willson et al., 1981.) However, some of the missing starspot energy in red dwarfs may be finding its way into the corona (Gershberg, 1983).

Active region areas on the surfaces of dMe stars may contribute to the differences between dM and dMe spectroscopic properties. With this assumption, Giampapa (1980) has estimated area coverage factors for several M dwarfs; in the most active dMe stars, coverage may be much more extensive than that in dM stars.

By analogy with the Sun, spots and active regions on M dwarfs are believed to be of magnetic origin. The surest evidence for magnetic fields in flare stars is the presence of large circular polarization in radio emission (Gibson, 1983), particularly when the maximum radio emission coincides with maximum visibility of a starspot (Linksy and Gary, 1983). The degree of circular polarization can be especially high during flares—essentially 100 percent (e.g., Lang et al., 1983). Indirect estimates of magnetic field strengths  $|B|$  based on interpretation of radio and X-ray properties (with guidance from solar analogs) have yielded values of  $|B|$  of at least 210 to 2960 gauss in a group of five M dwarfs (Golub, 1983), 1000 to 2000 gauss for UV Ceti (Linksy and Gary, 1983), and  $>9000$  gauss for YZ CMi (Haisch, 1983). Although these results are model-dependent, they suggest that fields of many kilogauss may exist on the surfaces of M dwarfs (cf. Mullan, 1984a). Direct measurements of field strength and areal coverage in M dwarfs by the Robinson (1980) technique have not yet been made.

Magnetic fields of kilogauss strength certainly interfere with convective flow patterns in M dwarfs. A steady-state model for a vertical flux tube in the umbra of a sunspot or starspot can be constructed by quantifying the

magnetic reduction in convective efficiency, allowing for finite electrical conductivity (Mullan, 1974a, 1974b). The missing thermal flux is converted into Alfvén waves, and a self-consistent depth-dependent model of a spot can be derived. The surface cooling can be severe ( $\sim 2000$  K on a red dwarf with surface field of  $\sim 20$  kilogauss). (See also Staude, 1978.) In normal conditions, the Alfvén waves are trapped beneath the surface of the spot by reflection at the steep photospheric density gradient. However, in exceptional conditions (during large flares), the Alfvén waves may be able to leak upward into the corona and escape from the star. Waves which have all of the properties predicted for such umbral Alfvén waves in sunspots have been detected recently in the solar wind following a large flare (Mullan and Owens, 1984), but there seems to be little likelihood of detecting umbral Alfvén waves in spots on M dwarfs.

The time-dependent behavior of magnetic flux tubes in red-dwarf convection zones may be more complex than in the solar case (because spot sizes on red dwarfs are much larger fractions of the stellar radius), and some of the solar analogs may therefore be invalid. For example, long-lived sunspots with well-developed penumbrae may owe their quasi-stability to the particular radial profile of differential rotation,  $\partial\Omega/\partial r$ , beneath the Sun's visible surface (Meyer et al., 1977). In the case of red dwarfs,  $\partial\Omega/\partial r$  may be quite different, and flux tubes may never achieve even a quasi-stable state; in that case, the assumption of a solar-like (umbral and penumbral) single spot may be irrelevant (Mullan, 1983; Vogt, 1983).

However, from the scale sizes of inhomogeneities on M dwarfs (both dark and bright; cf. Bopp, 1974a, 1974b), the magnetic flux ropes must be very large, of the order of a stellar radius,  $R_s$ , in diameter. Some measure of interference between magnetic fields and red-dwarf luminosity seems therefore likely. Bolometric variability due to such interference has been discussed by Hartmann and Rosner (1979) and by Gershberg (1978, 1983). It is not yet clear whether it is the total magnetic flux

in a tube which controls the variability, or the field strength.

Parenthetically, we note that, if a flux rope intersects the stellar surface over a length scale of  $\sim R_s$ , then the flux loops in the rope must arch up to heights which cannot be much less than  $\sim R_s$  above the photosphere. (In the solar case, coronal X-ray loops typically extend upward to less than  $\sim 0.1 R_s$ .) This quantitative feature should be borne in mind when one considers coronal heating in M dwarfs and flare visibility beyond the limb.

### Thermal Models of Internal Structure

Although the internal structure of a star is not the main topic of interest in this series of monographs, ultimately the most stringent test of a model atmosphere is: can it be matched with a sensible model of the *interior* of the star which it is supposed to represent? In other words, can complete stellar models be found which reproduce the observed mass-luminosity (M-L) relationship and the observed mass-radius (M-R) relationship on the lower main sequence? Several research groups have investigated these questions for stars of all spectral types along the main sequence. For present purposes, the most interesting result of these works is the prediction that a main-sequence star becomes completely convective if its mass is less than about  $0.3 M_\odot$ . (For a summary of this work, see Neece, 1984.) Because stars of such mass are predicted to have spectral type of middle M, the passage to complete convection is relevant to us here.

One of the parameters which characterizes convection in the thermal models described above (see the section *Thermal Model Atmosphere of Normal M Dwarfs*) is the mixing length,  $\alpha = L/H_p$ . The best-fitting models required  $\alpha \approx 1$ . However, Cox et al. (1981) attempted to integrate inward to the center of a star for which the surface parameters were fairly well known (60 Kruger A, mass  $0.27 M_\odot$ , luminosity  $3.81 \times 10^{31}$  ergs s<sup>-1</sup>,  $T_{\text{eff}} = 3100$  K). The only free parameter in the integration

is the value of  $\alpha$ : the requirement for a sensible model is that the integration must reach the center of the star with the proper mass. The star chosen by Cox et al. is particularly interesting because its mass falls close to the boundary where stars are expected to become completely convective. Thus, it might provide a rather stringent test of the convection theory. Surprisingly, Cox et al. found that, to obtain a sensible model, they had to assume  $\alpha = 0.07$  to  $0.17$  in regions in which the local temperature lay below 9000 K. (The range of values of  $\alpha$  corresponds to a variety of compositions and opacities.) These values of  $\alpha$  are much smaller than those used by the atmospheric modelers (Hershey, Mould, etc.), and they had the effect of making the convection zone quite shallow. The interior model of 60 Kruger A turned out to have an appreciable radiative core, with a mass of as much as  $0.62 M_*$  in one model. Of course, for a small enough mass, the models of Cox et al. would eventually have found a completely convective star. But the difference from earlier models of a nearly convective star was striking.

Cox et al. proposed the following explanation for small  $\alpha$ : magnetic fields in the envelope are expected to reduce the efficiency of convection on a global scale (see the section *Quasi-Thermal Model Photospheres of dMe Stars*), and the mixing-length formalism responds to this by reducing  $\alpha$ . Cox et al. (1981) claimed that the differences between their models and earlier ones are due to improved opacities and improved allowance for particle interactions (including electrostatic corrections) in the equation of state. Cox et al. claimed that their models were in better agreement with the M-L and M-R relationships than other models.

However, Neece (1984) subsequently used the same opacities and an improved equation of a state to derive models of lower main-sequence stars and found acceptable fits to the observed M-L and M-R relationships by assuming a conventional value for  $\alpha$ ,  $\alpha = 1$ . Neece found that complete convection sets in for stars with masses between 0.25 and  $0.3 M_\odot$ . In the somewhat more massive stars,  $0.4$



to  $0.55 M_{\odot}$ , Neece found that a small convective core appeared within the radiative interior. The major difference between the equation of state used by Cox et al. (1981) and by Neece occurs in Neece's inclusion of interactions between neutral particles and all other species (including molecules), rather than simply the electrostatic corrections. If this is really the only difference between the two modeling efforts, it suggests that the equation of state will have to be known with extraordinary precision if we are to obtain credible models of M dwarfs. On the other hand, VandenBerg et al. (1983) were able to reproduce the lower main sequence without having to assume a complicated equation of state.

Refinements of the equation of state have recently become a matter of pressing interest in solar physics in attempts to fit the rich spectrum of eigenmodes detected in solar oscillation studies. For example, electrostatic corrections to the equation of state increase the local gas pressure by up to 7 percent in models of the convection zone, push the base of the model convection zone deeper, and shift the model eigenfrequencies by amounts which are larger than the current error bars (Shibahashi et al., 1983). Cox et al. (1981) admitted that even the Coulomb corrections to the pressure in M dwarfs are very uncertain. Shibahashi et al. claim that the major unsolved problem in the solar model at present is how to calculate the internal partition functions for partially ionized constituents: these functions affect the degree of ionization of the gas by an amount which is serious enough that no known solar convection model satisfies all the constraints imposed by the observed eigenmode spectrum. If such problems exist in constraining solar models, the achievement of successful interior models of M dwarfs (in which partially ionized constituents are more abundant than in the Sun) is probably a matter of many years hence, even with purely thermal models.

In fact, Neece (1984) is sufficiently pessimistic about current uncertainties in the surface

parameters of M dwarfs (implying large uncertainties in interior structure) that he suggests we can no longer rely on data from single M dwarfs to improve our knowledge of the inner structure. Neece proposes that a more fruitful avenue of research may center on the cataclysmic variables (CV's); i.e., binaries in which an M dwarf is losing mass to a compact companion. The evolution of such a system depends on the way in which the M dwarf radius evolves with respect to the Roche lobe. It is interesting to note that, during mass loss, a convective star tends to expand, whereas a radiative star tends to contract. (This may be relevant to an understanding of the "gap" which exists in the CV period distribution between 2.0 and 3.0 hours: Robinson et al., 1981.) Thus, if a reliable evolutionary model of CV secondaries could be derived from the available CV data, it might lend some credibility to the interior models of M dwarfs.

However, even if thermal models can be derived with reliability, it will still be necessary to pick the surface boundary condition with care in order to get onto the correct adiabat. The extent to which this boundary condition depends on the overlying chromosphere and corona has not yet been determined. (In fitting the solar oscillation data, a rather realistic model chromosphere must be incorporated because the eigenmodes are standing waves in a cavity, one boundary of which is the chromosphere/corona.) As has been the case with the solar/stellar connection in the past, improvements in the equation of state in the solar context are expected to be beneficial to modelers of M dwarfs.

In regard to the oscillations, it is currently unknown to what extent energy transport in either g-modes or p-modes affects the internal structure of the Sun. In M dwarfs, the same problem exists except that, in the latest M dwarfs in which the radiative core presumably disappears, the g-modes will no longer contribute to the problem.

## NONPHOTOSPHERIC MODELS OF M DWARF ATMOSPHERES

All of the model atmospheres described previously are strictly photospheric: their emergent fluxes are adequate to account for the observed continua, plus the major molecular absorption features. The  $T(\tau)$  relations in all cases are monotonic. However, these models are incapable of accounting for the most characteristic feature of dMe spectra—the profiles of *emission* lines of hydrogen (and calcium). In fact, the profiles of even some of the strong atomic *absorption* lines such as  $H\alpha$  cannot be reproduced satisfactorily, particularly in the cores of the lines. Because the *wings* of  $H\alpha$  in the warmer M dwarfs have a contribution from the photosphere, purely photospheric models can achieve a measure of success in accounting for the wings of M dwarfs with  $T_{\text{eff}} \gtrsim 4000$  K (see Hershey, 1968). However, in the cooler M dwarfs in which photospheric contributions to  $H\alpha$  are expected to become vanishingly small, there is a major difficulty in explaining the presence of strong  $H\alpha$  absorption in such stars.

The first step in remedying these deficiencies has been to construct semiempirical chromospheres in which a temperature rise is superimposed above the photosphere and the temperature is adjusted until a satisfactory fit to some spectral feature is achieved. This approach is summarized for the nonflaring chromosphere in the section *Semiempirical Chromospheric Models*, for nonflaring transition regions in the section *Semiempirical Transition Region and Coronal Models*, and for flares in the section *Semiempirical Flare Models*.

A more fundamental approach seeks to predict mechanical fluxes from first principles and to use these predictions to compute chromospheric and coronal temperatures *ab initio*. (See the section *Nonthermal Models of M Dwarf Atmospheres*.) This approach would also be ultimately desirable for flares, but no such work has yet been published.

## Semiempirical Chromospheric Models

Kandel (1967b) studied a chromosphere with two temperature plateaus: the cooler ( $T_c = 3750$  K) radiates mainly in  $H^-$ , while the hotter radiates mainly by means of H atoms. Two parameters characterize a model:  $N_c$ , the electron column density above the base of the hot plateau (i.e., the chromosphere), and  $T_c$ , the temperature of the hot plateau. In each model, a two-level Ca atom was used to compute total emission fluxes in the K core. Comparing with available data, only a very restricted portion of ( $T_c, N_c$ ) space is acceptable: thus, if  $T_c = 2 \times 10^4$  K,  $\log N_c$  must lie in the range 18.9 to 19.1. Kandel's discussion of Balmer line emission, although inadequate (as he himself admitted), was important in stressing that, in red dwarfs, collisional processes might dominate over photoionization in the source function (contrary to the solar case).

This point was discussed in more detail by Fosbury (1974) in the context of an isothermal chromosphere. He found that, in an M dwarf with  $T_{\text{eff}} = 3500$  K, collisional control becomes strong enough to drive  $H\alpha$  into emission if  $N_c > 10^{11} \text{ cm}^{-2}$ . Fosbury suggested that a breakdown in the observed correlation between  $T_{\text{eff}}$  and  $H\alpha$  - width at  $T_{\text{eff}} \sim 3000$  K (Spinrad, 1973) can be interpreted in terms of a transition from photoelectric control to collisional control in the  $H\alpha$  source function. Thus, if chromospheric heating rates remain unchanged along the main sequence, the Balmer lines would inevitably go into emission at the coolest  $T_{\text{eff}}$ . This is consistent with the early work of Joy and Abt (1974), who found that, among M dwarfs, the fraction of stars classified as dMe (indicating Balmer emission) increases toward later types, reaching almost 100 percent at M5.5. Some exceptions to this rule may now exist (Giampapa, 1983), indicating that chromospheric heating may not in all cases remain unchanged in the coolest dwarfs. In the M dwarfs, irradiation by the overlying corona may dominate the heating of the chromosphere (Cram, 1982). The presence of weak X rays in

the coolest M dwarfs (Golub, 1983) undoubtedly contributes to weakening of Balmer emission there (Mullan, 1984b). However, even if  $H\alpha$  never goes into emission in certain cool dwarfs, remaining always an absorption line, this may still require significant chromospheric heating (Cram and Mullan, 1979).

Gershberg (1974) used a thick slab isothermal chromospheric model to interpret Balmer decrements in flare stars. The Sobolev method was used to determine the population of 30 levels in hydrogen. Only collisional processes were included; no photospheric radiation was included. A three-parameter family of models was considered:  $T_e$ ,  $n_e$ , and  $\beta_o$  (the photon escape probability at the center of Lyman- $\alpha$ ). Ratios of  $H\alpha/H\beta$  intensities are shown in Figure 10-4. The decrement may be either normal or inverted; both types of decrements are observed in flare stars. In quiescent conditions, Gershberg found best fits to 17 spectrograms of nine flare stars with  $n_e = (1-4) \times 10^{12} \text{ cm}^{-3}$ ,  $T_e = 10^4 \text{ K}$ , and  $\beta_o = (1-2) \times 10^{-6}$ . Such escape probabilities can be understood if there are velocity fields in the stars of the order of 10 to 30 km/s.

Subsequently, Grinin (1979) extended Gershberg's (1974) work to include the photospheric radiation field. Although this did not change the level populations by much (cf. Fosbury, 1974), so that the  $n_e$  and  $T_e$  values remained almost the same as above, the ex-

istence of the background radiation allowed Grinin to derive the fractional area of the disk covered by  $H\alpha$  emitting regions; he found 6 to 7 percent (AD Leo) and 14 percent (EQ PegA). Similar coverage was deduced by Greenstein (1977) in another flare star. Active region coverages of a few percent (< 6 percent) have also been derived for Proxima Centauri on the basis of a static-loop analysis of the X rays in the quiescent corona (Haisch et al., 1983). Extreme inversions in the Balmer decrement may indicate highly inhomogeneous physical conditions in the atmosphere (Grinin, 1980).

Kelch et al. (1979) modeled Ca K emission-line profiles in two cool dwarfs (61 Cyg B and EQ Vir) in terms of a chromospheric temperature profile,  $dT/d \log m = C_i$  ( $m$  is mass column density;  $C_i$  are constants, and  $i = 1, 2, \dots$ ). Several segments of chromosphere are adopted to fit various parts of the profile. Thus, the deepest part begins at  $\log m = +0.3$  (61 Cyg B) and  $-0.2$  (EQ Vir), where the temperature is allowed to depart from radiative equilibrium (RE), falling off less slowly with increasing height than in an RE model. At a certain point ( $\log m = -1.4$ , both stars), the temperature is allowed to pass through a minimum ( $T_{\min} = 0.8 T_{\text{eff}}$ ). Above  $T_{\min}$ ,  $T$  rises to 8000 K at the "top" of the chromosphere ( $m_o$ ). Above  $m_o$ ,  $T$  jumps abruptly to coronal values. In this structure, the radiative transfer code for a three-level  $\text{Ca}^+$  atom is solved until a best-fit profile is obtained.

The  $\text{Ca}^+$  data constrain the model mainly in the region close to  $T_{\min}$ ; above  $T_{\min}$ , in EQ Vir, for example, the best model has  $\gamma (= dT/d \log m) = 1800$  to 1900 K. This is much steeper than in the quiet Sun ( $\gamma = 900$  to 1000 K), but is comparable to active regions and flares ( $\gamma = 1600$  K). In less active stars, (e.g., 61 Cyg B),  $\gamma$  is closer to solar values. Kelch et al. suggest that dMe stars may differ from dM stars (i.e., have larger  $\gamma$ ) because of stronger magnetic fields (cf. also Mullan, 1975). The "best" models of Kelch et al. require chromospheric mechanical fluxes  $F_{\text{chr}}$  such that  $\phi_{\text{chr}} = F_{\text{chr}}/\sigma T_{\text{eff}}^4$  is  $6 \times 10^{-5}$  in the "quiet" star (61 Cyg B) and  $2 \times 10^{-4}$  in the active star (EQ

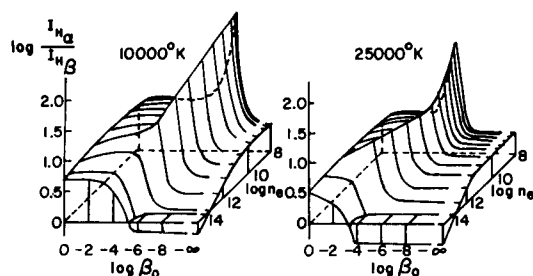


Figure 10-4. Relative intensities of  $H\alpha$  and  $H\beta$  in isothermal gas of temperature  $T_e$  and electron density  $n_e$ . Escape probability of a photon at the center of Lyman- $\alpha$  is denoted by  $\beta_o$  (Gershberg and Shnol', 1974).

Vir). (These ratios were estimated by summing over the strongest emission lines.) Although the models of Kelch et al. (1979) are subject to non-uniqueness problems, they had significant success in "predicting" that H $\alpha$  is in absorption in 61 Cyg B but in emission in EQ Vir, in qualitative agreement with observations. Although the H $\alpha$  profiles did not fit the data to better than a factor of 2, the work of Kelch et al. represented a start in studying nonlocal thermodynamic equilibrium (NLTE) radiative transfer in a realistic hydrogen atom, in cool dwarfs.

Independently, Cram and Mullan (1979) undertook a parametric study of Balmer line formation in the quiescent spectrum of cool dwarfs. A temperature rise of the form  $dT/d \log m = C_{1,2}$  in two segments (from  $T_{\min}$  to  $T_0$  and  $T_0$  to  $T_1$ ) was superimposed on Mould's (1976) photospheres. An H atom with five bound levels plus continuum was used, allowing the radiative-transfer problem to be solved in the Lyman continuum and in H $\alpha$ , H $\beta$ , and H $\gamma$ . The parameter space of the study is listed in Table 10-1, and representative H $\alpha$  profiles are shown in Figure 10-5. In the absence of a chromosphere (Model 1), H $\alpha$  would be only a weak absorption line in a star with  $T_{\text{eff}} = 3500$  K. As  $T_0$  increases and the chromosphere "builds up," H $\alpha$  first goes more deeply into *absorption*, reaching a maximum equivalent width (EW) of  $-0.66 \text{ \AA}$ : interestingly, 61 Cyg B, which has  $T_{\text{eff}}$  close to 3500 K, has H $\alpha$  in absorption with EW of just this order (Kelch et al., 1979). The implication is that, in cool dwarfs, even if H $\alpha$  is observed to be in *absorption*, there may be a well-developed chromosphere requiring significant mechanical energy deposition, and such stars may even be flare stars (e.g., SZ UMa; see Worden et al., 1981). In view of this, one should not conclude *solely on the basis of H $\alpha$  emission or absorption* that division of M dwarfs into dMe and dM is *necessarily* a division into "chromospheric" and "nonchromospheric" stars.

Cram and Mullan (1979) found that further buildup of the superposed chromosphere drives H $\alpha$  into emission, first in the wings and subsequently in the integrated profile. (A central

dip in the H $\alpha$  emission is caused by optical depth effects; early observational interpretations of such dips in terms of Zeeman splitting have not been confirmed.) For  $T_0 > 9700$  K, Cram and Mullan found that integrated H $\alpha$  is in emission (with  $\tau > 1$ ) for  $n_e(m_0) > 10^{11} \text{ cm}^{-3}$ , in good agreement with Fosbury's (1974) conclusion. Young et al. (1984) have used the results of Cram and Mullan to argue that the chromospheres in dM stars are systematically less dense than those in dMe stars. (For a two-component model, this conclusion is equivalent to varying areal coverage by active regions.) The H $\alpha$  profile is sensitive mainly to atmospheric behavior between temperatures of  $\sim 5000$  and  $\sim 30000$  K. Near the temperature minimum, the radiation field from the photosphere dominates the source function, and as a result, H $\alpha$  contains essentially no information about such regions. Thus, H $\alpha$  can serve as a useful complement to Ca K emission as a diagnostic of chromospheric structure in red dwarfs. In general, Mg h and k emission is most sensitive to conditions around 8000 K (i.e., around  $m_0$ ; see Kelch et al., 1979).

Recently, Linsky et al. (1982) have quantified some properties of dM and dMe chromospheres and have estimated mechanical energy fluxes,  $F_{\text{chr}}$ , in M dwarfs. They find that the fraction,  $\phi_{\text{chr}}$ , remains roughly constant among the main-sequence dM stars ( $\phi_{\text{chr}} \approx 2 \times 10^{-5}$ ), but in dMe stars,  $\phi_{\text{chr}}$  is about 5 times larger than in dM stars ( $\phi_{\text{chr}} \approx 10^{-4}$ ). However, the important point is that the dM stars *need* a finite amount of mechanical energy to create the chromosphere which gives rise to their H $\alpha$  lines in absorption; thus, one *cannot* conclude that chromospheres are entirely absent from dM stars. (The value of  $\phi_{\text{chr}}$  in the dM stars is close to the value in the quiet Sun.) Furthermore, whereas Mg II emission is the dominant line for chromospheric radiative losses in solar-type stars (and also in dM stars), in the dMe stars, emissions in Ca II and Fe II become comparable to Mg II, and Balmer line emission is several times stronger than Mg II. Optical depth in H $\alpha$  is certainly larger than

**Table 10-1**  
**Some Properties of Model dM Stellar Chromospheres\***

Model	$T_0$	$\log m_0$	$\log m_1$	$\log N_e$	$\tau_{H\alpha}$	EW <sub>H<math>\alpha</math></sub>	EW <sub>H<math>\beta</math></sub>	EW <sub>H<math>\gamma</math></sub>
1	- - - isothermal - - -			10.0	10	-0.08	-0.07	-0.07
2	5000	-5.5	-7.0	10.3	5	-0.60	-0.24	-0.13
3	7000	-5.5	-7.0	10.5	8	-0.66	-0.30	-0.16
4	9000	-5.5	-7.0	10.6	40	-0.62	-0.38	-0.26
5	9500	-5.5	-7.0	11.0	70	-0.22	-0.23	-0.16
6	10000	-5.5	-7.0	11.2	120	0.69	0.36	0.34
7	10500	-5.5	-7.0	11.5	200	3.68	1.76	1.74
8	9000	-5.0	-7.0	11.0	96	0.38	0.18	0.19
9	9000	-4.75	-7.0	11.3	140	2.24	1.14	1.17
10	9000	-4.5	-7.0	11.6	210	6.66	3.55	3.76
11	9000	-4.25	-7.0	11.9	290	15.18	8.34	9.34
12	9000	-5.0	-6.5	11.0	96	0.54	0.26	0.26
13	9000	-5.0	-5.5	11.1	85	0.73	0.42	0.42
14	9000	-5.0	-7.0	11.3	180	1.43	0.80	0.80
15	9000	-5.0	-7.0	11.2	150	1.58	0.86	0.87
16	9000	-5.0	-7.0	11.5	110	-0.08	-0.20	-0.17

\*Notes:

Models 1 to 14 are based on Mould's (1976) model with  $T_{\text{eff}} = 3500$  K,  $\log g = 4.75$ , and "old disk" abundances.

In model 14, the temperature minimum has been moved deeper, to  $T_{\text{min}} = 3081$  K and  $\log M_{\text{min}} = 0.56$ . In models, 1 to 13, the corresponding values are  $T_{\text{min}} = 1659$  and  $\log m_{\text{min}} = -0.65$ .

Model 15 is based on the same  $T_{\text{eff}}$  and  $\log g$  as above, but the relative abundances have been reduced by a factor of 5.

Model 16 has  $T_{\text{eff}} = 4200$  K,  $\log g = 4.75$ , and "old disk" abundances.

The value of  $T_1$  is always 30000 K.

$N_e$  is the maximum chromospheric electron density ( $\text{cm}^{-3}$ ).

EW is the equivalent width, negative for absorption lines. Units are in Å.

unity in dMe stars (Petterson, 1984). Hydrogen atom emissions are the dominant coolants of dMe chromospheres, although the calcium infrared triplet is also important in the most active stars (Petterson, 1984).

#### Semiempirical Transition Region and Coronal Models

The work of Linsky et al. (1982) also attempts to model the transition region (TR) between chromosphere and corona. They find that the fraction  $\phi_{TR}$  of  $\sigma T_{\text{eff}}^4$  which heats the

TR in dMe stars is as large as  $2 \times 10^{-5}$  in active stars, which is  $\sim 100$  times larger than  $\phi_{TR}$  in solar plages. X-ray data (Golub, 1983) can be interpreted to show that the coronal heating fraction in M dwarfs,  $\phi_{cor}$ , may also rise to a maximum of  $>100$  times larger than that in the Sun; in the strongest emitters (of spectral type M1-M2), the X-ray luminosity is  $\sim 1$  percent of the bolometric luminosity (i.e.,  $\phi_{cor} = 0.01$ ). This is the maximum value of  $\phi_{cor}$  reported for any single star of late spectral type. (In the Sun, the maximum value of  $\phi_{cor}$  in active regions is  $\sim 10^{-4}$  (Withbroe and Noyes,

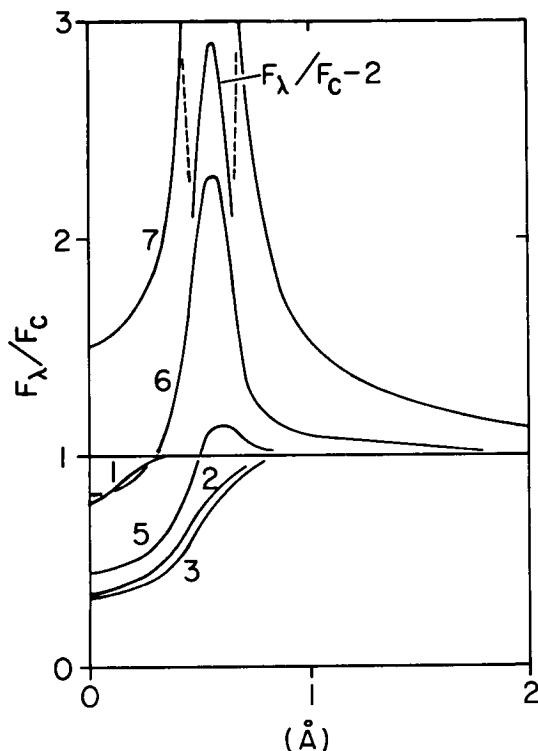


Figure 10-5. Relative flux profiles for the  $H\alpha$  line emitted by a grid of dM star chromospheres. The number associated with each profile refers to a model in Table 10-1. Note that only one half of each profile is shown (Cram and Mullan, 1979).

1977).) It is remarkable that, in M dwarfs, the nonthermal processes have an efficiency as large as 1 percent. The value of  $\phi_{cor}$  falls off rather steeply in the later M dwarfs (Mullan, 1984b). Now, in the quiet Sun,  $\phi_{chr} \sim 10 \phi_{cor}$ , whereas in active regions,  $\phi_{chr} \sim 2 \phi_{cor}$  (Withbroe and Noyes, 1977). Hence, the chromospheric heating problem to be solved by solar theorists is more exacting than the coronal heating problem. However, in certain M1-M2 dwarfs, it appears that  $\phi_{cor} > \phi_{chr}$  by a factor which may be as large as  $\sim 100$ , and the heating problem is qualitatively reversed. (At some spectral types, the X-ray luminosity,  $L_x$ , spans a range of up to 3 orders of magnitude, with no clear correlation with kinematic grouping; it seems unlikely that rotation differences alone can explain the large range in  $L_x$  (Johnson,

1983).) In dMe stars, the principal problem is to explain how the corona of certain M1-M2 dwarfs can be heated so efficiently (Mullan, 1984b). When this problem is solved, the chromospheric heating may actually be understood simply in terms of absorption of X rays from the overlying corona (Cram, 1982).

Why should coronal heating be so efficient in dMe stars? Gershberg (1983) has pointed out that, in certain flare stars,  $\phi_{cor}$  is comparable to the “missing flux” in starspots. He postulates that dMe stars are somehow efficient at transferring the “missing flux” into coronal heating (unlike the Sun). We will return to a quantitative discussion of this point in the section *Nonthermal Models of M Dwarf Atmospheres*.

### Semiempirical Flare Models

In the same spirit as the foregoing discussion, flare models have been proposed in which it is assumed that an abrupt energy release from an unspecified source causes local heating; the aim is then to reproduce flare light curves and colors as a time-dependent problem.

Gershberg (1970) has summarized the “nebular” model, in which a single impulse heats a region of the atmosphere to “chromospheric” temperatures (i.e.,  $(10-20) \times 10^3$  K). (These temperatures are constrained by the observed Balmer jumps.) Hydrogen recombination is assumed to be the source of flare light. As the flare volume expands, an analytic expression can be derived for the decaying branch of the flare light curve. Observed light curves were fitted quite well in many cases, and from the decay time, initial densities were derived:  $n_0 = 10^9$  to  $10^{11}$  cm $^{-3}$ . Subsequently, these estimates were found to be too small on two counts: they would require the flare volume to be extremely large (covering the entire stellar disk), and they would yield incorrect Balmer decrements (Gershberg, 1978). Apparently, flare decay is not due to recombination, but is controlled by some other slower process (Gershberg, 1975); Gershberg did not identify what that slower process might be.

A further argument against recombination as the source of flare light is based on the colors ( $U-B$ ,  $B-V$ ) of flare light; during flare decay, the predicted colors have a trajectory in the two-color diagram which is inconsistent with observations. However, it must be admitted that arguments based on the two-color diagram are not especially forceful; the observed colors are scattered over a wide area of the diagram, and many different interpretations of such colors are now known to be admissible. For example, Gershberg (1978, his Figure 2) shows how the observed  $U-B$ ,  $B-V$  colors in a large sample of flares (at maximum light) are spread over such a large area of the two-color diagram that individual events can be found which are consistent with one or more of seven different emission mechanisms:

1.  $H^-$  emission at  $T = (5-10) \times 10^3$  K
2. Nonthermal bremsstrahlung
3. Inverse Compton scattering of photospheric radiation
4. Synchrotron emission with spectral index 0-2
5. Optically thin plasma at  $T > 8 \times 10^4$  K
6. Blackbody with  $T > 4000$  K
7. Hydrogen plasma, optically thick in Balmer lines, with  $T = (15-25) \times 10^3$  K

The flare material considered by Gershberg is strictly "chromospheric," with no attempt to link it with the overlying corona or underlying photosphere. Kunkel (1970) proposed that the chromospheric flare would irradiate the photosphere and "burn" it to a state of a few hundred degrees hotter than the undisturbed photosphere. Kunkel suggested that the "burn" might help to interpret flare colors during the late stages of flare decay. That a flare can cause excess heating even in the photosphere has been established in the case of the Sun:  $\Delta T \sim 100$

K at  $\tau \sim 0.1$  during certain flares (Machado and Linsky, 1975).

The most detailed discussion of the photospheric "burn" has been given by Grinin (1973, 1976) in his treatment of time-dependent radiative transfer in the presence of an external source of radiation. Scattering and diffusion of a  $\delta$ -function source causes the reflected signal to have a well-defined form with a time scale of the order of radiative relaxation time,  $\tau_r$ . If the flare light has a particular time variation (e.g.,  $L(t) \sim (1 + t/B)^{-2}$ , as the "nebular" model predicts), the reflected light can be derived by convolution. For conditions in M dwarfs,  $\tau_r \sim 3$  to 5 minutes. Since this is comparable to observed flare decay times, Grinin concludes that the reflected signal may indeed contribute detectably to flare light.

An interesting feature which emerges from Grinin's work concerns the temperature sensitivity of the opacity in the photosphere. Depending on the atmospheric composition (especially the metal abundance) when the photosphere is "burned," the opacity may increase momentarily so much that the "burn" actually blocks off the radiation emerging from inside the star. Grinin (1976) proposes that "negative" flares may be due to this effect (i.e., dips in the star's light below the nonflaring level) occurring during a flare. Although many features of "negative" flares can apparently be accounted for in this context, Grinin pointed out that the model cannot explain "negative" dips which occur *before* the optical flare itself, unless there is some other source of radiation (other than optical flare light) to cause the prior "burn." Grinin did not specify such a source. However, his analysis of the evolution of flare colors during decay has already led him to conclude that the "burn" must be much hotter than the photosphere (by several thousand degrees); he realized that this would be almost impossible if "nebular" gas alone were the source of the "burn." Something hotter would be required. Subsequently, Grinin (1980) proposed a different explanation of "negative" flares in which preflare changes in magnetic fields reduce the active region emission. A different

interpretation of "negative" flares is provided by Giampapa et al. (1982a); they suggest that a flare occurs when a prominence is destabilized and deposits its material on the surface of the star. They suggest that, before hitting the surface, the prominence material obscures part of the surface, thereby causing a "negative" precursor to the main flare outburst.

A further indication of the important role which deep-seated gas has on stellar flares emerged from the work of Grinin and Sobolev (1977). They placed the optical flare-emitting gas in the photosphere/chromosphere boundary region ( $N = 10^{15}$  to  $10^{17}$  cm $^{-3}$ ) and considered H $^-$  emission as well as opacity beyond the Balmer jump. This model successfully accounted for the *UBV* colors of flare light, the correlation with *U-B* color, and the Balmer jump. Solar "white" light flare emission may also emerge from analogously deep parts of the atmosphere. Nevertheless, the relaxation time scales associated with deep gas are difficult to reconcile with observed light curves, and the necessity of an externally imposed time scale again emerges if such light curves are to be understood. Gershberg (1978) has argued that, since flare luminosity  $L$  on many different flare stars (with quite different photospheric properties) obeys a "universal" relation near maximum ( $d \log L/dt_{\max} \sim L^{-1/4}$  (Shakhovskaya, 1974a, 1974b), this in itself is not compatible with significant photospheric contributions to flare light near maximum. Gershberg believes that a "universal" relation is more likely to be an indication that very hot gas is contributing to flares at maximum. (See also Mullan, 1976a, 1976b.)

Cram and Woods (1982) have reported a systematic study of semiempirical flare models along similar lines to those used by Cram and Mullan (1979) for quiescent chromospheres. (The models were chosen to represent various proposed heating mechanisms; an analogous study of solar flares has recently been reported by Canfield et al. (1984).) In the flare case, however, as well as computing Balmer line pro-

files, the broadband continuum was also computed. A general conclusion is that the flaring chromosphere has higher pressures and temperatures than the quiescent chromosphere. In regards to high pressures in solar flares, Canfield et al. (1984) have shown that flares which show H $\alpha$  in emission without any central reversal must have  $P \gtrsim 100$  dyn/cm $^2$  (i.e.,  $10^2$  to  $10^3$  times the quiet Sun value). On the other hand, flares which show broad Stark wings in H $\alpha$  cannot be understood in terms of high pressure alone; they require large fluxes of energetic electrons  $> 10^{10}$  erg/cm $^2$ /sec in electrons with  $E > 20$  keV. Empirical evidence for high pressure in line-forming regions of stellar flares indeed emerges from a study of the broadening of the various lines in the Balmer series; in six flares on YZ CMi, the data yield electron densities in a rather well-constrained range:  $10^{13}$  to  $10^{14}$  cm $^{-3}$  (Worden et al., 1984). If  $T = 10^4$  K in the Balmer line-forming region, this indicates pressures of 30 to 300 dyn/cm $^2$  in these flares. A significant conclusion which emerges from the work of Cram and Woods is that, in order to produce strong narrow Balmer lines with an inverted decrement but at the same time show no very strong Balmer jump (such as flare data suggest), it is necessary to have the line-emitting region cover at least 10 to 20 percent of the stellar surface (i.e., much larger than previous estimates of flare area). If the area of the flare decreases with time, while leaving pressures and temperatures high, the observed time dependence of the Balmer decrement can be explained. On the other hand, the flare *continuum* (as opposed to lines) appears to emerge from much smaller areas ( $\sim 1$  percent of the surface). (Mochnecki and Zirin (1980) derived flare areas in YZ CMi and UV Cet which are equivalent to  $< 0.2$  percent areal coverage of the surface in continuum emission; these estimates are uncertain because of the assumption that the flare radiates as a blackbody.) Cram and Woods mention the analog of solar flares in which compact "kernels" of continuum emission involve deep atmospheric heating in a few isolated patches under the H $\alpha$  flare. The areas of H $\alpha$  flare emission proposed by Cram



and Woods (1982) are reminiscent of the areal coverage of starspots on red dwarfs. If the spots are indeed magnetic flux tubes, it appears that the entire tube must participate in the release of magnetic energy when a flare occurs.

As was mentioned above, the  $H\alpha$  line profile is sensitive to conditions even at rather high temperatures (30000 K or even more; cf. Dame and Cram, 1981). Other arguments (Gershberg, 1970; Grinin, 1976) also suggest that, in order to obtain a correct interpretation of flare light, it is important to consider the effects of very hot gas (i.e., much hotter than "nebular").

Andrews (1965) was the first to model flare light curves by free-free emission (implying very hot gas,  $T > 10^7$  K). This emission gives way to recombination as the temperature drops, and Andrews successfully modeled observed light curves by this two-stage process. At the time of Andrew's work, there were no data to prove the existence of hot gas in stellar flares. Since 1975, abundant evidence has accumulated from various satellites (Astronomical Netherlands Satellite, Small Astronomy Satellite C, High Energy Astronomy Satellite 1, and Einstein) to confirm Andrews' assumption of gas at  $T \sim$  few times  $10^7$  K (cf. Haisch, 1983).

Mullan (1976b) investigated the effects of inserting a plasma at  $T = 10^7$  K into a flare star atmosphere. The flare plasma establishes a thermal structure which, if steady state is achieved, has one of two universal forms, depending on whether the density is constant or the pressure is constant (Shmeleva and Syrovatskii, 1973). The constant density limit is relevant in "spike" flares, in which energy is released so rapidly ( $< 1$  minute) that pressure equalization is not immediately possible. In such cases, continuum emission would dominate the flare spectrum as observed (Moffett, 1973). The  $U-B$ ,  $B-V$  colors of free-free continuum are consistent with flare colors. In M dwarfs, the background photosphere is not bright enough to mask the plasma continuum, although in the solar case, this would not in general be true. If stellar flare light is indeed

free-free emission from coronal plasma, the bolometric corrections to  $UBV$  flare luminosities may be as large as  $\sim 5^m$  (cf. also Kodaira, 1977); this is the major source of uncertainty in our current knowledge of stellar flare energetics. (Of course, the bolometric corrections to the photospheric radiation of a flare star is also large, up to  $5^m$  in the latest M dwarfs (Petterson, 1983), but these bolometric corrections are easier to determine because of the nontransient nature of the emission.)

Eventually, the flare plasma reaches pressure equalization, and then the thermal structure of the flare atmosphere switches to the second universal form described by Shmeleva and Syrovatskii (1973). The result is two-fold: the flare emission falls dramatically (by a factor of  $10^2$  to  $10^3$ ), and the continuum gives way to emission lines. Flares in which the energy release is slow should evolve entirely via the constant pressure structure and should show predominantly line emission throughout, rather than in the decay state only; this is consistent with data (Moffett, 1973).

Hot plasma loses energy by radiation, conduction, and expansion. Estimates (Mullan, 1976a) suggested that conductive time scales would be the most rapid in stellar flares. Thus, a flare light curve would be the response of the atmosphere heated from above mainly by conduction. This led to the prediction that the optical flare light curve decays on a time scale which is associated with conduction from the flare plasma. Thus, the unidentified "slowly decaying agent" which Gershberg (1975) invoked to account for optical flare light curves was attributed to conduction. This conclusion is model-dependent because of uncertainties in the length scales associated with conduction. Note that, in his analysis of X-ray data, Haisch (1983) assumes that flare plasma cools not only by conduction but also by radiation, with equal cooling rates in both modes.

Flare plasma irradiates the photosphere with X rays which, as they propagate through the

denser gas in the lower atmosphere, become degraded into optical photons (Mullan and Tarter, 1977). These optical photons, in conjunction with the direct emission from the flare plasma, may help to explain the evolution of flare light in the  $U-B$ ,  $B-V$  plane. Also, the fact that X rays can reach the photosphere essentially instantaneously, whereas conductive heating requires a finite time scale, may help to explain why "negative" flares in Grinin's (1976) scenario can occur prior to the optical flare.

An idealized time-dependent model of flare plasma, including energy losses by radiation, conduction, and expansion, was calculated by Mullan (1977). Nonmonotonic light curves, including preflares and still-stands, can be reproduced in this model. In subsequent applications of these calculations to flares on AD Leo, temperatures of  $(20-30) \times 10^6$  K were derived, with coronal densities of  $(0.35-1) \times 10^{12}$  cm $^{-3}$  (Schneeberger et al., 1979); in the case of a flare on YZ CMi, the derived temperatures were  $13 \times 10^6$  K, with densities of  $0.07 \times 10^{12}$  cm $^{-3}$  (Worden et al., 1984). The flare temperatures derived by this method are quite consistent with those derived in other flares by more direct methods (Haisch, 1983), whereas the densities are not grossly different from those derived by Gershberg (1978) in his "nebular" model.

Kodaira (1977) incorporated very hot gas into his "reservoir-emitter" model of a very large stellar flare on EV Lac. The hottest gas ( $T \sim 10^8$  K) in the magnetic flux-tube reservoir emits X rays, whereas optical flare light comes from "emitter" gas at the foot points of the flux tube ( $T \sim 10^5$  K). To contain the reservoir,  $B = 200$  gauss is required, corresponding to 2 kilogauss at the foot points. The latter cover  $\sim 10$  percent of the stellar hemisphere, comparable to what Cram and Woods (1982) also require. The "reservoir" flux tube extends to great heights; Kodaira finds that its scale length is comparable to the radius of the star. This conclusion is consistent with the point noted above: if starspots and active regions occupying  $\sim 10$  to 20 percent of the disk area are

magnetic flux tubes, then these tubes must extend upward to great heights ( $\sim R_s$ ) above the photosphere. Flux-tube lengths of up to  $(0.5-1) R_s$  have also been inferred in the case of a flare on Proxima Cen and a giant flare in a Hyades star (Haisch, 1983).

In view of the possibility that magnetic fields in red dwarfs may be considerably stronger than those in the Sun, we may ask why the flare temperatures in the red dwarfs are essentially identical to solar flare temperatures. One possibility is to hypothesize that the atmospheric densities scale as  $n \sim B^2$  (Mullan, 1975); if a flare involves magnetic reconnection in which the final plasma has a beta ( $= P_{gas}/P_{mag}$ ) of order unity (Moore and Datlowe, 1975), the flare temperatures would be relatively constant if  $n \sim B^2$ . Is it reasonable to expect such a scaling? To answer that, we note that, in the solar corona, static loops of length  $L$  and pressure  $p$  have temperatures  $T$  which scale as  $T \sim (pL)^{1/3}$  (Rosner et al., 1978). Observed loop temperatures are confined within a fairly narrow range so that, to a first approximation, we can adopt  $T \approx \text{constant}$ . Hence, the densities  $n$  scale as  $L^{-1}$ . Now, the vertical field strength,  $B_z$ , in a coronal loop has been found to scale as follows:  $p \sim B_z^{3/2} L^{-1/4}$  (Golub et al., 1982). If  $T = \text{constant}$ , this reduces to  $n \sim B_z^2$ . This can be considered as providing some support for Mullan's (1975) hypothesis.

The most detailed study to date of the hydrodynamics of a stellar flare is that of Livshits et al. (1981). A beam of nonthermal electrons is supposed to be created (in an unspecified manner) in the atmosphere, and a one-dimensional numerical code is used to follow the effects. When the electrons deposit their energy, a region of high pressure is formed: shocks propagate upward and downward. Radiative losses are especially severe in the downward shock, and as a result, that shock may have a very large density jump ( $\rho_2/\rho_1 \sim 100$ ). Because of the greater densities and shorter scale heights in red-dwarf atmospheres, the region of shock compression may become optically thick even in the visible continuum ( $\tau_{4500} \sim 1$ )

for a short time. The authors ascribe the short-lived continuum in red-dwarf flares to this process. This treatment of a shock in a stellar atmosphere is more realistic than an earlier discussion by Korovyakovskaya (1972); in the latter case, although flare light curves could be reproduced, there were difficulties with the energetics.

Nonthermal electrons also play an essential role in the "fast-electron hypothesis" of stellar flares (Gurzadyan, 1973). There, flare light is supposed to be the result of an inverse Compton scattering of photospheric photons off a large number of 1- to 10-MeV electrons. However, there are many serious difficulties with this hypothesis (Kodaira, 1977; Gershberg, 1978). For example, to explain a total flare energy of  $10^{34}$  ergs, allowing for the inefficiency of Compton scattering ( $\sim 10^{-5}$ ), Gurzadyan's model requires that the total energy in fast electrons must be  $10^{39}$  ergs. If each electron has an energy of  $\sim 1$  MeV, this requires a total number of  $10^{45}$  electrons. With a flare duration of  $\sim 10^2$  seconds and even supposing the electron beam is spread out over the maximum possible area ( $\pi R_s^2$ , where  $R_s$  is the stellar radius), the current density would still be so large that a magnetic field of  $\sim 10^{13}$  gauss would be induced by the beam. (Note that there is no reverse current to cancel the outward-streaming electrons.) The back-reaction of such a field on the star would be enormous. These numbers are so large that his hypothesis must be rejected.

### Nonthermal Models of M Dwarf Atmospheres

Kuperus (1965) and de Loore (1970) discussed the generation of acoustic waves in convection zones, followed by their dissipation in the atmosphere. However, neither of these authors considered stars as cool as M dwarfs. Renzini et al. (1977) extended the earlier work to M dwarfs. They estimated that, at  $T_{\text{eff}} = 3200$  K and  $\log g = 5$ , the acoustic flux  $F_{ac} = 140 \text{ ergs cm}^{-2} \text{ s}^{-1}$  (i.e., some  $10^5$  times less than solar). Bohn (1981) discusses various correc-

tions to this estimate and finds that  $F_{ac}$  may be  $10^2$  to  $10^3$  times larger than Renzini et al. found. However, even this correction is not large enough to avoid the prediction that  $F_{ac}$  in red dwarfs should be *less* than in the Sun: the data do not support this. Thus, the flux of chromospheric mechanical energy,  $F_{chr}$ , alone in the dMe stars is about equal to the solar value (Linsky et al., 1982), whereas the coronal radiative losses,  $\phi_{cor}$  in certain dMe stars is  $> 100$  times the solar value (Mullan, 1984b). To these, the effects of the mechanical energy required to power the winds from the stars should be added for completeness. Therefore, if acoustic waves are to provide for the sum total of mechanical energy deposition in the atmosphere, we must have  $F_{ac} = F_{tot}$ , where  $F_{tot} = F_{chr} + F_{cor} + F_{wind}$ . However, in the solar corona, even in coronal holes, where  $F_{wind}$  is maximum,  $F_{chr}$  dominates  $F_{wind}$  by a factor of almost 10. In solar active regions,  $F_{wind}$  contributes less than 1 percent to  $F_{tot}$ . (See, for example, Withbroe and Noyes, 1977.) Hence, in the solar case, neglect of  $F_{wind}$  appears to be permissible in estimating  $F_{tot}$ .

What can we say about M dwarfs? It would seem that  $F_{wind}$  will also be rather unimportant for two reasons: (1) the gravitational potential well of an M dwarf is deeper than solar, thereby impeding steady wind flow; and (2) strong coronal radiation from M dwarfs suggests that their surfaces are almost entirely covered with active regions (Mullan, 1984b), and from these, steady wind flow is also inhibited by closed magnetic field lines. We cannot exclude the possibility that transient mass ejections occur during flares (Coleman and Worden, 1976; Shlosman et al., 1979). It seems acceptable to discuss  $F_{tot}$  outside flares as the sum of only two terms,  $F_{chr}$  and  $F_{cor}$ . If we then attempt to equate  $F_{tot}$  with  $F_{ac}$ , we find that the predicted behavior of convective acoustic power along the lower main sequence is not even qualitatively in agreement with the observed properties of chromospheres/coronae. Adding the possible effects of transient mass ejections during flares makes the disagreement even worse. Therefore,

either acoustic heating is unimportant or perhaps the wrong velocity has been used in estimating acoustic power. (See the section *Quasi-Thermal Model Photospheres of dM Stars*.)

Ulmschneider and his colleagues have devoted considerable attention to the propagation and dissipation of acoustic waves in stellar atmospheres. On the basis of this, Ulmschneider et al. (1977a, 1977b) have found that, in a red dwarf with  $T_{\text{eff}} = 4000$  K,  $\log g = 4$ , a shock should form (and presumably lead to a temperature minimum) at a mass loading  $m_s \sim 10^{-3}$  gm cm $^{-2}$ . However, empirically, the temperature minimum in such a star is observed to occur much deeper, at  $m_s = 0.05$  gm cm $^{-2}$  (Cram and Ulmschneider, 1978; Kelch et al., 1979). Again, the acoustic theory does not appear to apply well to red dwarfs.

The deficiency of acoustic heating has led several investigators to consider how magnetic effects might enter into the mechanical heating of red-dwarf atmospheres. Katsova (1973) considered the dissipation of slow-mode waves in a stratified atmosphere. With a prescribed flux at the bottom of the atmosphere, and allowing the dissipation to be balanced locally at each height by radiative losses, Katsova obtained the run of temperature with density (i.e., height). The magnetic field was set to a constant value (20 gauss). Two quasi-plateaus appear in the atmosphere at  $\sim 10^4$  K and at  $\sim 5 \times 10^4$  K. The calculations were carried up to a maximum temperature of  $2 \times 10^6$  K. At that level, Katsova found that the density in the red dwarf was  $\sim 10$  times larger than in the solar model at the same temperature. Glebocki et al. (1974) extended Katsova's work to somewhat warmer stars. For semiquantitative arguments in support of magnetic heating of the chromospheres/coronae of cool dwarfs, see Mullan (1975). For a discussion of propagation and dissipation of magnetohydrodynamic waves in stellar atmospheres in general, see Leibacher and Stein (1981) and Ulmschneider and Stein (1982); on the basis of the latter work, Marcy (1984) has argued that chromospheric/coronal heating in G and K dwarfs can be understood in terms of slow and Alfvén modes. However, this argu-

ment has not yet been extended to M dwarfs, where serious line blending has so far prevented meaningful application of Marcy's observing technique.

Much detailed research has been carried out in the post-Skylab era in an attempt to put magnetic heating in the solar corona on a sound footing. The incentive for this is the obvious dominance of magnetic loops in the solar corona. Current dissipation and heating by Alfvén waves have received the most attention. However, thus far, none of these ideas have been applied in detail to red dwarfs, apart from the application of certain scaling relations between loop lengths, pressures, temperatures, field strengths, etc. (Golub, 1983; Haisch, 1983). Even in the absence of detailed application, some points can already be made. For example, if the densities in red-dwarf chromospheres/coronae are indeed  $\geq 10$  times solar (cf. Gershberg, 1970; Katsova, 1973; Mullan, 1975), then the critical current,  $j_c = nev_{th}$ , where anomalous current dissipation sets in (i.e., when the drift velocity equals the thermal speed) is  $> 10$  times larger in red dwarfs than in the Sun. As a result, the critical Joule dissipation rate,  $\sim j_c^2/\sigma$ , is  $> 100$  times greater than solar. If the onset of rapid Joule heating when the current goes supercritical acts as a flare initiator, then this result may help to explain why many stellar flares have a very rapid rise to maximum light, particularly the so-called "spike" flares (Moffett, 1973), in which rise times are  $< 1$  second.

As a second example, we may consider the heating of coronal magnetic loops by waves. The waves are presumably generated over a certain spectrum of frequencies in the convection zone. There is no a priori reason why the wave spectrum in the convection zone should be well matched to resonance periods of atmospheric loops. In the Sun, the convective wave periods are in the region of  $\tau_c \sim 500$  seconds (Ionson, 1982), whereas typical loop resonance periods may be of order  $\tau_A = 5$  seconds (Mullan, 1984b). Thus, in the Sun, impedance matching between the source of the waves and their sink is not particularly good, and the electrodynamic

coupling efficiency,  $\epsilon$ , between convection zone and corona ( $\sim \tau_A/\tau_c$  when  $\tau_A < \tau_c$ ; Ionson, 1984) is small, of the order of 1 percent. As a result, the mechanical energy deposited in solar coronal loops is quite small (typically no more than  $10^{-4}$  of the radiant energy flux).

However, consider how these arguments are affected in the case of red dwarfs. Their convection time scales,  $\tau_c \sim T/\mu g v_c$  (where  $T$  is the temperature,  $\mu$  is molecular weight,  $g$  is gravity, and  $v_c$  is convective velocity), are certainly *shorter* than solar time scales (see Figure 10-6). Estimates of  $\tau_A$  in red dwarfs are difficult to make, but the observational evidence which is currently available (Mullan, 1984b) indicates values of  $\tau_A$  as shown in Figure 10-6 (i.e., tending toward *greater* values in later spectral types). The possibility of a "crossover point," where  $\tau_c = \tau_A$ , in Figure 10-6 is of great interest. If such a crossover were to exist, the coupling efficiency,  $\epsilon$ , would approach unity (i.e.,  $\sim 10^2$  times larger than in the Sun). In fact, M1-M2 dwarfs are observed to have coronal heating efficiencies which are  $\gtrsim 10^2$  times solar (Mullan, 1984b). The crossover depicted schematically in Figure 10-6 is meant to suggest that spectral types of M1-M2 ( $T_{\text{eff}} \approx 4000$  K) are in fact a reasonable location for  $\tau_c \approx \tau_A$ . Moreover, at spectral types later than M1-M2, impedance matching once more breaks down, and the coupling efficiency,  $\epsilon$ , should fall off rapidly; this agrees with the behavior of the observed X-ray fluxes.

The question of magnetic heating of even quiescent atmospheres of red dwarfs requires much more work to be put on a quantitative basis. *A fortiori*, the relationship of magnetic heating to stellar flares is even more complicated, and no work has apparently been done on this topic. It may be that, if "normal" heating can be accounted for, then some flares can be considered as simply an extreme version of coronal heating or vice versa (cf. Mullan, 1979; Doyle and Butler, 1985; Butler and Rodonò, 1985). (Where does one draw the boundary between a small flare and a bright active region loop, for example?) On the other hand, the impulsive phase of flares may require a

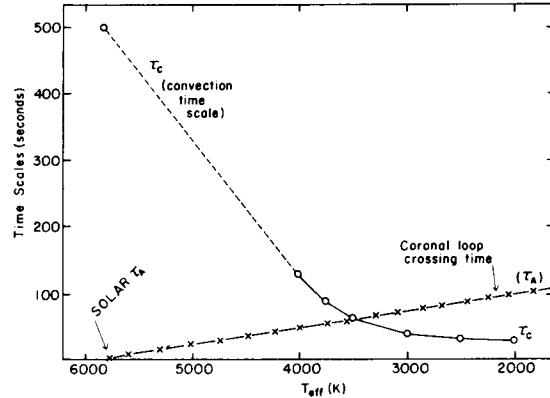


Figure 10-6. Time scales that enter into Ionson's (1984) discussion of coronal heating:  $\tau_c$  = convection time;  $\tau_A$  = Alfvén crossing time in a loop. Note that  $\tau_c$  decreases at later spectral types, while  $\tau_A$  may increase. At the crossover ( $\tau_c = \tau_A$ ), coronal heating is maximally efficient (Mullan, 1984b).

qualitatively distinct process, perhaps arising from an explosive instability. In this regard, we note that, in flare star coronae, the electron densities derived from the conductive interpretation of flare light curves (Mullan, 1976a) are apparently low enough that certain plasma instabilities should indeed proceed explosively (Spicer, 1975) rather than "slowly."

## REFERENCES

- Andrews, A.D. 1965, *Irish Astron. J.*, 7, 20.
- Auman, J.R. 1969, *Astrophys. J.*, 157, 799.
- Bohn, H.U. 1981, in *Second Cambridge Workshop on Cool Stars, Stellar Systems, and the Sun*, ed. M. Giampapa and L. Golub, SAO Special Report 391, Vol. 1, p. 67.
- Bonsack, W.K., and Culver, R.B. 1966, *Astrophys. J.*, 145, 767.
- Bopp, B.W. 1974a, *Mon. Not. Roy. Astr. Soc.*, 166, 79.

- Bopp, B.W. 1974b, *Mon. Not. Roy. Astr. Soc.*, **168**, 255.
- Bopp, B.W., and Espenak, F. 1977, *Astron. J.*, **82**, 916.
- Butler, C.J., and Rodono, M. 1985, Armagh Observatory Preprint, No. 9.
- Canfield, R.C., Gunkler, T.A., and Ricchiazzi, P.J. 1984, *Astrophys. J.*, **282**, 296.
- Carbon, D.F. 1979, *Ann. Rev. Astron. Astrophys.*, **17**, 513.
- Chan, K., and Sofia, S. 1984, *Astrophys. J.*, **282**, 550.
- Cloutman, L.D. 1979, *Astrophys. J.*, **227**, 614.
- Coleman, G.D., and Worden, S.P. 1976, *Astrophys. J.*, **205**, 475.
- Cox, A.N., Shaviv, G., and Hodson, S.W. 1981, *Astrophys. J. (Letters)*, **245**, L37.
- Cram, L.E. 1982, *Astrophys. J.*, **153**, 768.
- Cram, L.E., and Mullan, D.J. 1979, *Astrophys. J.*, **234**, 579.
- Cram, L.E., and Ulmschneider, P. 1978, *Astron. Astrophys.*, **62**, 239.
- Cram, L.E., and Woods, T.D. 1982, *Astrophys. J.*, **257**, 269.
- Dame, L., and Cram, L.E. 1981, *Bull. Amer. Astron. Soc.*, **13**, 820.
- de Loore, C. 1970, *Astrophys. Space Sci.*, **6**, 60.
- Doyle, J.G., and Butler, C.J. 1985, *Nature*, **313**, 378.
- Fosbury, R.A.E. 1974, *Mon. Not. Roy. Astr. Soc.*, **169**, 147.
- Gershberg, R.E. 1970, *Flares on Red Dwarf Stars*, in Russian (Moscow: Nauka), English translation by D.J. Mullan (Armagh Observatory).
- Gershberg, R.E. 1974, *Soviet Astron. AJ*, **18**, 326.
- Gershberg, R.E. 1975, in *Proc. IAU Symp. 67, Variable Stars and Stellar Evolution*, ed. V.E. Sherwood and L. Plaut (Dordrecht: Reidel), p. 47.
- Gershberg, R.E. 1977, *Peremennye Zvezdy*, **20**, 283.
- Gershberg, R.E. 1978, *Low Mass Flare Stars*, in Russian (Moscow: Nauka).
- Gershberg, R.E. 1983, in *Proc. IAU Colloq. 71, Activity in Red Dwarf Stars*, ed. P.B. Byrne and M. Rodonò (Dordrecht: Reidel), p. 493.
- Gershberg, R.E., and Shnol', E.E. 1974, *Izv. Krim. Astrofiz. Obs.*, **50**, 122.
- Giampapa, M.S. 1980, in *Cool Stars, Stellar Systems and the Sun*, ed. A.K. Dupree, SAO Special Report 389, p. 119.
- Giampapa, M.S. 1983, in *Proc. IAU Symp. 102, Solar and Stellar Magnetic Fields: Origins and Coronal Effects*, ed. J.O. Stenflo (Dordrecht: Reidel), p. 187.
- Giampapa, M.S., Africano, J.L., Klimke, A., Parks, J., Quigley, R.J., Robinson, R.D., and Worden, S.P. 1982a, *Astrophys. J. (Letters)*, **252**, L39.
- Giampapa, M.S., Worden, S.P., and Linsky, J.L. 1982b, *Astrophys. J.*, **258**, 740.
- Gibson, D.M. 1983, in *Proc. IAU Colloq. 71, Activity in Red Dwarf Stars*, ed. P.B. Byrne and M. Rodonò (Dordrecht: Reidel), p. 273.

- Glebocki, R., Sikorski, J., and Barylko, J. 1974, *Acta Astron.*, **24**, 343.
- Golub, L. 1983, in *Proc. IAU Colloq. 71, Activity in Red Dwarf Stars*, ed. P.B. Byrne and M. Rodonò (Dordrecht: Reidel), p. 83.
- Golub, L., Noci, G., Poletto, G., and Vaiana, G.S. 1982, *Astrophys. J.*, **259**, 359.
- Greenstein, J.L. 1977, *Pub. Astron. Soc. Pacific*, **89**, 304.
- Grinin, V.P. 1973, *Izv. Krim. Astrofiz. Obs.*, **48**, 58.
- Grinin, V.P. 1976, *Izv. Krim. Astrofiz. Obs.*, **55**, 179.
- Grinin, V.P. 1979, *Izv. Krim. Astrofiz. Obs.*, **59**, 154.
- Grinin, V.P. 1980, in *Flare Stars, FUORS, and Herbig-Haro Objects* (in Russian), ed. L.V. Mirzoyan (Byurakan Ap. Obs.), p. 23.
- Grinin, V.P., and Sobolev, V.V. 1977, *Astrofizika*, **13**, 587.
- Gurzadyan, G.A. 1973, *Flare Stars*, in Russian (Moscow: Nauka).
- Haisch, B.M. 1983, in *Proc. IAU Colloq. 71, Activity in Red Dwarf Stars*, ed. P.B. Byrne and M. Rodonò (Dordrecht: Reidel), p. 155.
- Hartmann, L., and Rosner, R. 1979, *Astrophys. J.*, **230**, 802.
- Heney, L.G., Vardya, M.S., and Bodenheimer, P. 1965, *Astrophys. J.*, **142**, 841.
- Hershey, J.L. 1968, *Ph.D. Dissertation* (Charlottesville: Univ. of Virginia).
- Ionson, J. 1982, *Astrophys. J.*, **254**, 318.
- Ionson, J. 1984, *Astrophys. J.*, **276**, 357.
- Johnson, H.M. 1983, *Astrophys. J.*, **273**, 702.
- Joy, A.H., and Abt, H.A. 1974, *Astrophys. J. Supplement*, **28**, 1.
- Kandel, R. 1967a, *Annales d'Astrophys.*, **30**, 439.
- Kandel, R. 1967b, *Annales d'Astrophys.*, **30**, 999.
- Katsova, M.M. 1973, *Astron. Zh.*, **50**, 774 (*Soviet Astron. AJ*, **17**, 492).
- Kelch, W.L., Linsky, J.L., and Worden, S.P. 1979, *Astrophys. J.*, **229**, 700.
- Kodaira, K. 1977, *Astron. Astrophys.*, **61**, 625.
- Korovyakovskaya, A.A. 1972, *Astrofizika*, **8**, 247.
- Kunkel, W.E. 1970, *Astrophys. J.*, **161**, 503.
- Kuperus, M. 1965, *Recherches Astr. Obs. Utrecht*, **17**, 1.
- Lang, K.R., Bookbinder, J., Golub, L., and Davis, M.M. 1983, *Astrophys. J. (Letters)*, **272**, L15.
- Leibacher, J., and Stein, R. 1981, in *Second Cambridge Workshop on Cool Stars, Stellar Systems, and the Sun*, ed. M. Giampapa and L. Golub, SAO Special Report 392, Vol. 1, p. 23.
- Lin, R.P., Schwartz, R.A., Kane, S.R., Pelling, R.M., and Hurley, K.C. 1984, *Astrophys. J.*, **283**, 421.
- Linsky, J.L., Bornmann, P.L., Carpenter, K.G., Wing, R.F., Giampapa, M.S., Worden, S.P., and Hege, E.K. 1982, *Astrophys. J.*, **260**, 670.
- Linsky, J.L., and Gary, D. 1983, *Astrophys. J.*, **274**, 776.

- Livshits, M.A., Badalyan, O.G., Kosovichev, A.G., and Katsova, M.M. 1981, *Solar Phys.*, **73**, 269.
- Machado, M.E., and Linsky, J.L. 1975, *Solar Phys.*, **42**, 395.
- Marcy, G. 1984, *Astrophys. J.*, **276**, 286.
- Meyer, F., Schmidt, H.U., and Weiss, N.O. 1977, *Mon. Not. Roy. Astr. Soc.*, **179**, 741.
- Mochnecki, S., and Zirin, H. 1980, *Astrophys. J. (Letters)*, **239**, L27.
- Moffett, T.J. 1973, *Bull. Amer. Astron. Soc.*, **5**, 399.
- Moore, R.L., and Datlowe, D. 1975, *Solar Phys.*, **43**, 189.
- Mould, J.R. 1976, *Astron. Astrophys.*, **48**, 443.
- Mould, J.R. 1978, *Astrophys. J.*, **226**, 923.
- Mould, J.R. 1980, private communication, quoted by VandenBerg et al., 1983.
- Mould, J.R., and Hyland, A.R. 1976, *Astrophys. J.*, **208**, 399.
- Mullan, D.J. 1974a, *Astrophys. J.*, **187**, 621.
- Mullan, D.J. 1974b, *Astrophys. J.*, **192**, 149.
- Mullan, D.J. 1975, *Astron. Astrophys.*, **40**, 41.
- Mullan, D.J. 1976a, *Astrophys. J.*, **207**, 289.
- Mullan, D.J. 1976b, *Astrophys. J.*, **210**, 702.
- Mullan, D.J. 1977, *Astrophys. J.*, **212**, 171.
- Mullan, D.J. 1979, *Irish Astron. J.*, **14**, 73.
- Mullan, D.J. 1983, in *Proc. IAU Colloq. 71, Activity in Red Dwarf Stars*, ed. P.B. Byrne and M. Rodonò (Dordrecht: Reidel), p. 527.
- Mullan, D.J. 1984a, *Astrophys. J.*, **279**, 767.
- Mullan, D.J. 1984b, *Astrophys. J.*, **282**, 603.
- Mullan, D.J., and Owens, A.J. 1984, *Astrophys. J.*, **280**, 346.
- Mullan, D.J., and Tarter, C.R. 1977, *Astrophys. J.*, **212**, 179.
- Neece, G.D. 1984, *Astrophys. J.*, **277**, 742.
- Nordlund A. 1976, *Astron. Astrophys.*, **50**, 23.
- Petterson, B.R. 1983, in *Proc. IAU Colloq. 71, Activity in Red Dwarf Stars*, ed. P.B. Byrne and M. Rodonò (Dordrecht: Reidel), p. 19.
- Petterson, B.R. 1984, *Astrophys. J.*, **282**, 214.
- Porter, J.G., Toomre, J., and Gebbie, K.R. 1984, *Astrophys. J.*, **283**, 879.
- Renzini, A., Cacciari, C., Ulmschneider, P., and Schmitz, F. 1977, *Astron. Astrophys.*, **61**, 39.
- Robinson, R.D. 1980, *Astrophys. J.*, **239**, 961.
- Robinson, E.L., Barker, E.S., Cochran, W.N., and Nather, R.F. 1981, *Astrophys. J.*, **251**, 611.
- Rosner, R., Tucker, W.H., and Vaiana, G. 1978, *Astrophys. J.*, **220**, 643.
- Schneeberger, T.J., Linsky, J.L., McClintock, W., and Worden, S.P. 1979, *Astrophys. J.*, **231**, 148.
- Shakhovskaya, N.I. 1974a, *Izv. Krim. Astrofiz. Obs.*, **50**, 84.
- Shakhovskaya, N.I. 1974b, *Izv. Krim. Astrofiz. Obs.*, **51**, 92.
- Shibahashi, H., Noels, A., and Gabriel, M. 1983, *Astron. Astrophys.*, **123**, 283.



- Shlosman, I., Kozlovsky, B.Z., and Shaviv, G. 1979, *Astron. Astrophys.*, **73**, 358.
- Shmeleva, O.P., and Syrovatskii, S.I. 1973, *Solar Phys.*, **33**, 341.
- Spicer, D.S. 1975, *Bull. Amer. Astron. Soc.*, **7**, 397.
- Spinrad, H. 1973, *Astrophys. J.*, **183**, 923.
- Spruit, H.C. 1982, *Astron. Astrophys.*, **108**, 348.
- Staude, J. 1978, *Bull. Astron. Inst. Czecho.*, **29**, 71.
- Tsuji, T. 1966, in *Colloq on Late Type Stars*, ed. M. Hack (Trieste: Osservatorio Astronomico di Trieste).
- Ulmschneider, P., Kalkofen, W., Nowak, T., and Bohn, U. 1977a, *Astron. Astrophys.*, **54**, 61.
- Ulmschneider, P., Schmitz, F., Renzini, A., Caccari, C., Kalkofen, W., and Kurucz, R. 1977b, *Astron. Astrophys.*, **61**, 515.
- Ulmschneider, P., and Stein, R. 1982, *Astron. Astrophys.*, **106**, 9.
- VandenBerg, D.A., Hartwick, F.D.A., Dawson, P., and Alexander, D.R. 1983, *Astrophys. J.*, **266**, 747.
- Vardya, M.S. 1966, *Mon. Not. Roy. Astr. Soc.*, **134**, 347.
- Vardya, M.S., and Böhm, K.-H. 1965, *Mon. Not. Roy. Astr. Soc.*, **131**, 89.
- Vogt, S.S. 1983, in *Proc. IAU Colloq. 71, Activity in Red Dwarf Stars*, ed. P.B. Byrne and M. Rodonò (Dordrecht: Reidel), p. 137.
- Willson, R.C., Gulkis, S., Jannsen, M., Hudson, H.S., and Chapman, G.A. 1981, *Science*, **211**, 700.
- Withbroe, G.W., and Noyes, R.W. 1977, *Ann. Rev. Astron. Astrophys.*, **15**, 363.
- Worden, S.P., Schneeberger, T.J., and Giampapa, M.S. 1981, *Astrophys. J. Supplement*, **46**, 159.
- Worden, S.P., Schneeberger, T.J., Giampapa, M.S., Deluca, E.E., and Cram, L.E. 1984, *Astrophys. J.*, **276**, 270.
- Young, A., Skumanich, A., and Harlan, E. 1984, *Astrophys. J.*, **282**, 683.